

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/318435667>

Generation of shock lamellae and melting in rocks by lightning-induced shock waves and electrical heating

Article in *Geophysical Research Letters* · July 2017

DOI: 10.1002/2017GL073843

CITATIONS

12

READS

307

4 authors:



Jiangzhi Chen

Institute of Deep-Sea Science and Engineering, Chinese Academy of Sciences

10 PUBLICATIONS 51 CITATIONS

SEE PROFILE



Chiara Elmi

James Madison University

30 PUBLICATIONS 85 CITATIONS

SEE PROFILE



David Goldsby

University of Pennsylvania

34 PUBLICATIONS 228 CITATIONS

SEE PROFILE



Reto Gieré

University of Pennsylvania

188 PUBLICATIONS 3,328 CITATIONS

SEE PROFILE

Some of the authors of this publication are also working on these related projects:



Formation of fulgurites and natural glasses [View project](#)



Gravity-driven lava flow [View project](#)

RESEARCH LETTER

10.1002/2017GL073843

Key Points:

- A cylindrical air dynamics model was employed to estimate the shock wave pressure caused by a lightning strike on rocks
- An idealized point current source model was employed to calculate the electrical heating and subsequent cooling of rock fulgurites
- Model results were compared with literature data collected on natural rock fulgurites to explain the existence of shock lamellae

Correspondence to:

J. Chen,
jiangzhi@sas.upenn.edu

Citation:

Chen, J., C. Elmi, D. Goldsby, and R. Gieré (2017), Generation of shock lamellae and melting in rocks by lightning-induced shock waves and electrical heating, *Geophys. Res. Lett.*, 44, doi:10.1002/2017GL073843.

Received 18 APR 2017

Accepted 7 JUL 2017

Accepted article online 14 JUL 2017

Generation of shock lamellae and melting in rocks by lightning-induced shock waves and electrical heating

Jiangzhi Chen¹ , Chiara Elmi¹ , David Goldsby¹ , and Reto Gieré¹
¹Department of Earth and Environmental Science, University of Pennsylvania, Philadelphia, Pennsylvania, USA

Abstract The very rapid energy release from impact events, such as those resulting from lightning strikes or meteorites, can drive a variety of physical and chemical processes which alter rocks and result in the formation of natural glasses (i.e., fulgurites and tektites). Fulgurite is the vitrified soil, sand, or rock resulting from lightning strikes. A thunderbolt is associated with air temperatures of up to 10^5 K, which can heat rocks to >2000 K within tens of microseconds. The rapid fusing and subsequent quenching of the surface of the rock leaves a distinctive, thin, garbled coating composed of a glassy to fine-grained porous material. Previous studies on rock fulgurites found planar deformation features in quartz crystals within the target rock substrate, evidence of strong shock waves during fulgurite formation. In this paper, we simulated the shock pressure and temperature caused by an idealized lightning impact on rocks and compared the model results with observations on rock fulgurites from the literature. Our model results indicate that a lightning strike can cause >7 GPa pressure on the rock surface, generate a layer of fulgurite (of radius ~ 9 cm), and leave a burned region (of radius ~ 11 cm). The fulgurites found on rock surfaces share many features with sand fulgurites, but their spatial distribution is completely different, as sand fulgurites are hollow tube structures. Our study on rock fulgurites provides an indirect constraint on the energy of a lightning event and also demonstrates that the presence of shock features in rocks cannot be taken as unequivocal evidence for impact events.

1. Background

A lightning strike is a complicated phenomenon composed of three stages. First, a “stepped leader” descends from the cloud and contacts an oppositely charged target (e.g., the ground, architectural monuments, or other structures). Second, a “dart leader” travels back to the cloud, followed by a return stroke, i.e., the main flash, which travels from the cloud and strikes the target again. The return stroke, with an energy density of 3.3 MJ/m within a channel of radius ~ 1 cm [Jones *et al.*, 1968], is associated with air temperatures of up to 10^5 K [Uman, 1964] and a current of up to 30 kA [Uman and Krider, 1989]. The overall energy of the lightning strike is estimated to be $10^9 - 10^{10}$ J [Rakov and Uman, 2007].

When lightning strikes a target, such as a rock, the target can be heated to over 2000 K within tens of microseconds [Uman and Krider, 1989]. The resulting vitrified soil, sand, or rock is called a fulgurite. Pasek *et al.* [2012] classified fulgurites into four types, depending on whether the lightning strikes sand, clay, or rock. Sand fulgurites are usually found in deserts, featuring subsurface, branching, and hollow glass tubes [Rakov and Uman, 2007]. Occasionally, sand fulgurites can reach several meters in length (e.g., one fulgurite found in Florida was ~ 5 m long [see Rakov and Uman, 2007]). Rock fulgurites, however, appear as a distinctive, thin, garbled coating composed of glassy to fine-grained porous material [Essene and Fisher, 1986; Grapes and Müller-Sigmund, 2010; Gieré *et al.*, 2015; Elmi *et al.*, 2017] and may include burned organic material present on the rock surface before lightning impact [Elmi *et al.*, 2017]. In some cases, fulgurites are found on man-made structures, such as windmills [Brocklesby, 1869, see pp. 148]. Fulgurites can also be formed artificially by high-voltage electrical arcing [Williams and Johnson, 1980; Uman and Krider, 1989; Kumazaki *et al.*, 1993; Pasek and Hurst, 2016]. Our model focuses on rock fulgurites, particularly, fulgurites from the granite surface at Les Pradals, France [Gieré *et al.*, 2015] (Figure 1b) and Mount Mottarone, Italy [Elmi *et al.*, 2017] (Figure 1c).

Besides high temperature, the lightning strike also generates a strong shock wave in the vicinity of the strike point, evidenced by the presence of shock lamellae in the target rock, the so-called “planar deformation

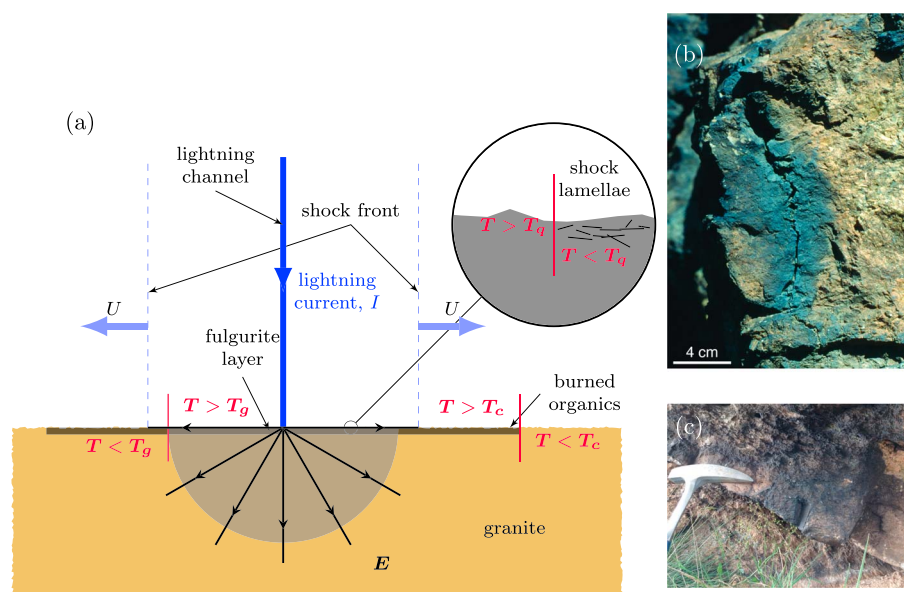


Figure 1. (a) Schematic of lightning hitting granite at 90° . The lightning causes a cylindrical shock wave in the air, leaving microscale shock features in the quartz crystals on the surface. Electrical heating causes the temperature to rise, melts the rock incongruently, and generates a thin fulgurite layer on the surface when the temperature is above the granite melting point. The organic material on the weathered granite surface within a certain distance of the strike point is burned and leaves a distinctive black mark. (b) Rock fulgurite from Les Pradals, France [Gieré *et al.*, 2015]. (c) Rock fulgurite from Mount Mottarone, Italy.

features" (PDFs) in crystalline quartz [Gieré *et al.*, 2015]. Shock lamellae are also associated with meteorite impacts [Bohor *et al.*, 1984; Goltrant *et al.*, 1992; Langenhorst and Deutsch, 1994; Alvarez *et al.*, 1995], but lightning impacts may also produce shock features in rocks [Gieré *et al.*, 2015]. Analysis of shocked crystalline quartz suggests shock pressures > 10 GPa [Grieve *et al.*, 1996] and sometimes ~ 22 – 30 GPa in magnitude [Langenhorst and Deutsch, 1994; Carter *et al.*, 2010a; Ende *et al.*, 2012]. However, the temperature rise associated with the lightning strike must be relatively moderate, likely lower than the quartz annealing point (1200°C [Grieve *et al.*, 1996]), otherwise the observed PDFs would disappear.

It must be emphasized that the transmission of energy from the lightning bolt to the rock during shock deformation and electrical heating, and the accompanying physical and chemical changes in the rock, absorb only a fraction of the total energy of the lightning, with the rest of the energy dissipated in the air in the form of thunder, heated air, light, and electromagnetic waves. The aim of this paper is to provide an idealized model of the shock wave, as well as the rapid electrical heating and subsequent cooling, in a granitic rock. The model estimates the pressure and temperature disturbance caused by a lightning strike and sheds light on the generation of shock lamellae and the energy transferred from the lightning to the target rock. The model is compared with field observations and experimental analyses of granite fulgurite samples from Gieré *et al.* [2015] and Elmi *et al.* [2017]. Using our model, we constrain various physical parameters associated with the lightning strike on rocks, including the energy density of the lightning bolt, and estimate the physical conditions and parameters controlling the formation of shock lamellae in fulgurites.

2. Model

We employed typical parameter values for the lightning and for the granite rock, as given in Table 1. The lightning is assumed to strike with a 90° angle of incidence with the rock surface (Figure 1a). We treat the shock wave around the lightning as being excited from a very rapid energy release from the lightning channel, which is treated as a line source. Estimates of the energy density of the lightning have a wide range of 10^3 to 10^5 J/m obtained from radiation, electrostatics, or optics [see Rakov and Uman, 2007, Table 12.1]. Collins *et al.* [2012] suggested a much higher energy density range, 10 – 50 MJ/m, based on estimates from arcing experiments. Pasek and Hurst [2016] concluded that lightning strikes with an energy density > 6 MJ/m may account for only 3–4% of all fulgurites formed annually. We take a value of 3.3 MJ/m after Jones *et al.* [1968]. The temperature of the lightning channel T_e is set to 10^5 K [Uman, 1964].

Table 1. Nominal Parameter Values of the Lightning and the Granite

Model Parameters				Value
Lightning	energy density	E	(J/m)	3.3×10^6
	channel radius	a	(m)	0.01
	peak current intensity	I_0	(A)	2×10^4
	rise time	t_r	(s)	10^{-6}
	total duration	t_m	(s)	50×10^{-6}
	decay constant	k	(s ⁻¹)	5×10^4
	temperature	T_e	(K)	3×10^4
Air	specific heat ratio	γ		1.4
	ambient temperature	T_a	(°C)	25
Rainfall	density	ρ_w	(kg/m ³)	1000
	specific heat	C_w	(J/(kg K))	4184
	precipitation rate	v	(cm/h)	0.76
	vaporization temperature	T_w	(°C)	100
	enthalpy of vaporization	$\Delta_v H$	(J/kg)	2.26×10^6
Granite	density	ρ	(kg/m ³)	2670
	specific heat	C	(J/(kg K))	790
	melting temperature	T_g	(K)	973
	thermal diffusivity	κ	(m ² /s)	0.913×10^{-6}
	electrical conductivity	σ	(S/m)	10^{-3}
Quartz	annealing temperature	T_q	(°C)	1200
Organic material	combustion temperature	T_c	(°C)	300

The air is treated as an isentropic ideal gas with constant specific heats C_p and C_v , and the specific heat ratio $\gamma = C_p/C_v = 1.4$. Rock surfaces are in general somewhat weathered before lightning strikes, with typically higher thermal and electrical conductivities in the weathered part than in the unaltered rocks due to the presence of moisture. However, *Elmi et al.* [2017] showed that the weathered layer on granite from Mottarone appears to be thinner than 1 mm, whereas the rock itself is tens of centimeters in thickness or larger. We treated the rock as a homogeneous half-space with uniform thermal and electrical properties. The thermal diffusivity κ of granite is 0.913 mm²/s, the specific heat C is 790 J/(kg K), and the density ρ is 2670 kg/m³ [Robertson, 1988]. The melting point of the granite T_g is dependent on the water content, and we used a value of 973 K for wet granite [Eppelbaum et al., 2014]. The annealing temperature T_q of quartz, beyond which the shock lamellae are destroyed, is ~ 1200 °C [Grieve et al., 1996]. The electrical conductivity σ of granite is taken as 10^{-3} S/m [Olona et al., 2010]. The parameters are treated as constants, but we further discuss alteration of air and rock properties at high temperatures and in strong electric fields below.

The combustion temperature of organic matter (e.g., lichens and other plant materials) on the rock surface is unknown. However, for wood, dehydration and charring commences at ~ 100 – 200 °C, whereas combustion occurs at ~ 200 – 300 °C [White and Dietenberger, 2001]. We assume a combustion temperature T_c of organic material of 300 °C. After the lightning strike and associated heat pulse, the rock surface cools down due to evaporation of rain drops and convection of heated air. Meteorological agencies usually use a rain water accumulation rate v of 0.76 cm/h as a boundary between moderate and heavy rain, which we adopt here as the precipitation rate, and the cooling rate per unit area due to rain water evaporation on the rock surface is $v\rho_w[C_w(T_w - T_a) + \Delta_v H]$ where the evaporation temperature $T_w = 100$ °C, and the enthalpy of vaporization of water at T_w is $\Delta_v H = 2.26$ MJ/kg [Marsh, 1987]. We ignore the advective heat transfer by rain water flowing over the rock surface, because significant water flow only exists after the rock surface is below T_w .

2.1. The Pressure Disturbance Caused by the Shock Wave

Gieré et al. [2015] reported that shock lamellae were only found within the top 3 μ m of the rock surface, suggesting that the most relevant pressure disturbance is from the shock wave around the lightning bolt. We ignore the bow shock at the tip of the lightning flash in our model, because the return stroke (main flash) travels at a speed of $c_i = 1 - 2 \times 10^8$ m/s [Rakov and Uman, 2007], greatly exceeding the speed of sound in air,

$c_s \approx 340$ m/s. After the rock is struck by the lightning, the trailing shock wave hits the surface, and body waves propagate inside the rock; however, no shocked quartz has yet been observed deep in the rock interior (i.e., >3 μm deep), indicating that the effect of these body waves can be neglected. The pressure from the magnetic field is $p_B = \mu_0 I^2 / (8\pi^2 r^2)$ around the line current I , where $\mu_0 = 4\pi \times 10^{-7}$ N/A² is the vacuum magnetic permeability. We ignore this pressure because it is only $\sim 10^4$ Pa at the radius of the lightning channel, which is much smaller than the shock pressure.

2.1.1. Cylindrical Shock Wave Around the Lightning

Lightning traverses in air can extend to tens or even hundreds of kilometers in length [Rakov and Uman, 2007], and the longest-distance flash reported traveled 321 km [Lang et al., 2016]. Until the lightning hits the ground, the radial shock wave can be treated as cylindrical, and the wave only propagates in the radial direction perpendicular to the lightning channel. Assuming that the air is an inviscid ideal gas, the Navier-Stokes equation with cylindrical symmetry is

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial r}, \quad (1)$$

where $u(r,t)$ is the air flow velocity, $p(r,t)$ is the pressure, and $\rho(r,t)$ is the air density. The continuity equation is

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} = -\rho \left(\frac{\partial u}{\partial r} + \frac{u}{r} \right), \quad (2)$$

and another governing equation is the isentropic relation for air

$$\frac{D}{Dt} \left(\frac{p}{\rho^\gamma} \right) = \left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \right) \left(\frac{p}{\rho^\gamma} \right) = 0. \quad (3)$$

The undisturbed air pressure is p_0 , the air density is ρ_0 , and $c_s = \sqrt{\gamma p_0 / \rho_0}$ is the speed of sound in undisturbed air. The shock wave has a radius R propagating at a speed $U = dR/dt$. From the Rankine-Hugoniot condition, the disturbed pressure p_1 , density ρ_1 , and velocity u_1 immediately behind the shock wave satisfy

$$\frac{\rho_1}{\rho_0} = \frac{\gamma - 1 + p_1(\gamma + 1)/p_0}{\gamma + 1 + p_1(\gamma - 1)/p_0} \approx \frac{\gamma + 1}{\gamma - 1} \quad (4)$$

$$\frac{U^2}{c_s^2} = \frac{\gamma - 1 + p_1(\gamma + 1)/p_0}{2\gamma} \approx \frac{p_1(\gamma + 1)/p_0}{2\gamma} \quad (5)$$

$$\frac{u_1}{U} = \frac{2(p_1/p_0 - 1)}{\gamma - 1 + p_1(\gamma + 1)/p_0} \approx \frac{2}{\gamma + 1} \quad (6)$$

with the approximations valid for strong shocks where $p_1/p_0 \rightarrow \infty$. Following the dimensional treatment of Taylor [1950a], the shock radius can be expressed as

$$R = S(\gamma) E^{1/4} \rho_0^{-1/4} t^{1/2} \quad (7)$$

where $S(\gamma)$ is a dimensionless function depending on the specific heat ratio γ only. We solve the governing equations (1) to (3) using the self-similar approach as in Lin [1954] (or see Appendix A), and the results are presented in Figure 2.

Numerical evaluations show that the shock radius is

$$R = \left(\frac{4E}{B\rho_0} \right)^{1/4} t^{1/2} \quad (8)$$

where the constant $B \approx 3.94$. The pressure behind the shock front is

$$p_1 = \frac{2E}{BR^2(\gamma + 1)} = 0.212ER^{-2}. \quad (9)$$

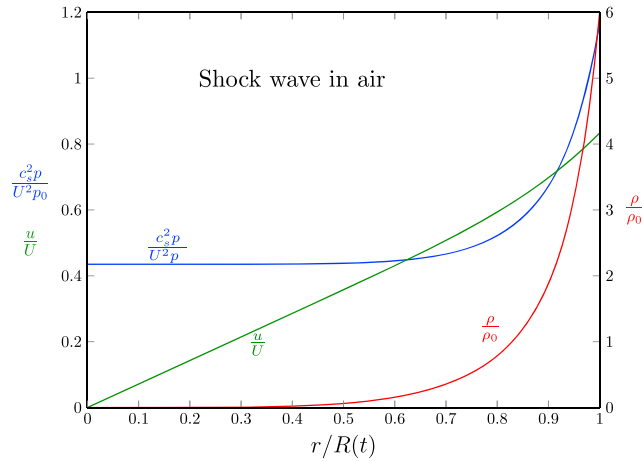


Figure 2. Normalized air pressure, normalized velocity, and normalized density as functions of radial position $r/R(t)$ inside the shock tube. Near the shock front, as r approaches R , both the pressure and density of air increase rapidly, whereas the velocity profile remains almost linear.

If we take the energy density $E = 3.3 \text{ MJ/m}$ [Jones *et al.*, 1968], and if $R \sim a = 1 \text{ cm}$ is the radius of the lightning channel, then the shock pressure is

$$p_1 \approx \frac{2E}{Ba^2(\gamma + 1)} = 7.0 \text{ GPa} \quad (10)$$

which is smaller than the pressure range ($\geq 10 \text{ GPa}$) required to generate shock lamellae in quartz [Grieve *et al.*, 1996]. However, if the energy density E is higher, for example, 10 MJ/m , the lower bound suggested by Collins *et al.* [2012], then $p_1 = 0.212ER^{-2}$ is large enough to generate a shock pressure of 21 GPa at the boundary of the lightning channel, sufficient to generate shocked quartz, and similar to pressures induced by meteorite impacts.

2.1.2. Shock Wave in the Rock

Due to the short duration of the lightning strike, the wave propagation in solid media resulting from the lightning strike can be treated as having been excited from a point explosion in an elastic half-space. As a simplified estimate, the pressure disturbance in the rock decays with distance R from the hit point as

$$p \sim 0.156 \frac{a^3}{R^3} \frac{2E}{Ba^2(\gamma + 1)}. \quad (11)$$

The R^{-3} falloff of pressure means that pressures in the interior of the rock will be far too small to form shocked quartz when R is greater than $\sim 4 \text{ mm}$. We also ignore strain heating here because the strain energy release and associated heating must be smaller than electrical heating due to the strike, as discussed below.

2.2. Thermal Evolution of the Rock During and After the Lightning

2.2.1. Electrical Heating

The electrical current through the rock is assumed to follow a pulse form $I(t) = I_0 \beta(t)$ [Plooster, 1971]

$$\beta(t) = \begin{cases} t/t_r, & \text{for } t < t_r \\ \exp[-k(t - t_r)], & \text{for } t_r < t < t_m \end{cases} \quad (12)$$

where the peak current intensity $I_0 = 20 \text{ kA}$, the decay constant $k = 5 \times 10^4 \text{ s}^{-1}$, and the duration $t_m = 50 \mu\text{s}$. The $\sim 1 \text{ cm}$ radius lightning channel is so narrow that it can be treated as a point current source. We set the origin of the cylindrical coordinate system at the lightning strike point, with the z axis pointed downward into the rock. Since the rock is assumed to be homogeneous, the current is evenly distributed in a half sphere of area $2\pi(r^2 + z^2)$, so that the electric field $\mathbf{E}$ only has a radial component

$$E_r = \frac{I}{2\pi\sigma(r^2 + z^2)}. \quad (13)$$

The electric field has a singularity at $r = z = 0$ due to the point current source treatment of the lightning. One way to remove this singularity is to use a lightning channel with a finite radius of ~ 1 cm. Then the electric field in the vicinity of the lightning channel is dominated by a vertical component

$$E_z \approx \frac{I}{\pi \sigma a^2}, \quad (14)$$

whereas for regions far from the lightning channel, the electric field is still E_r . These two expressions differ by a factor of 2 because the area changes from that of a circle, πa^2 , to that of a half sphere, $2\pi a^2$ at the channel boundary. A more detailed explanation is provided in Appendix B.

The temperature evolution during the lightning strike is given by

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T + \frac{\sigma}{\rho C} |\mathbf{E}|^2. \quad (15)$$

The boundary and initial conditions are

$$T \Big|_{r,z \rightarrow 0} \Rightarrow \text{finite}, \quad T \Big|_{r,z \rightarrow \infty} = T_a, \quad \kappa \rho C \frac{\partial T}{\partial z} \Big|_{z=0} = Q_v + Q_r, \quad T \Big|_{t=0} = T_a, \quad (16)$$

where the heat loss rate due to rainfall vaporization is $Q_v = v \rho_w [C_w(T_w - T_a) + \Delta_v H]$, and the heat radiation rate is $Q_r = \sigma_{\text{SB}} (T^4 - T_a^4)$ where $\sigma_{\text{SB}} = 5.67 \times 10^{-8} \text{ W/(m}^2 \text{ K}^4)$ is the Stefan-Boltzmann constant. We ignore the electromagnetic effects related to a time-dependent electric field and discuss the possible effects in later sections. The equation is inhomogeneous with a nonlinear boundary condition. During the lightning strike, the distance inside the rock over which heat conduction is significant is $\Delta r \sim \sqrt{\kappa t_m} \approx 7$ mm, and the rate of heat conduction is $\kappa \nabla^2 T \approx \kappa (T_e - T_a) / \Delta r^2$. Similarly, since $Q_r \sim \sigma_{\text{SB}} T_e^4 \gg Q_v$, the heat loss rate at the surface near the hit point is of order $\sigma_{\text{SB}} T_e^4 / (\rho C \Delta r)$. These two processes are negligible compared with the electrical heating rate $\sigma |\mathbf{E}|^2 / (\rho C)$. Therefore, the temperature after the lightning strike is dominated by electrical heating

$$T_i \approx \int_0^{t_m} \frac{\sigma}{\rho C} |\mathbf{E}|^2 dt + T_a. \quad (17)$$

This equation can be modified readily to remove the singularity at the origin and make the temperature continuous

$$T_i - T_a \approx \begin{cases} \frac{A I_0^2 t_m}{4 \pi^2 \rho C \sigma R^4}, & \text{for } R \geq a/\sqrt{2} \\ \frac{A I_0^2 t_m}{\pi^2 \rho C \sigma a^4}, & \text{for } R < a/\sqrt{2}, \end{cases} \quad (18)$$

where $R^2 = r^2 + z^2$ and the constant $A = t_m^{-1} \int_0^{t_m} \beta^2 dt \approx 0.205$. The temperature at the surface as a function of r can be compared with the size of the burned regions observed on fulgurite surfaces. The region within which the granite is expected to be partially melted is obtained by solving $T_i(r) = T_g$, which gives a radius of ~ 9.3 cm for fulgurite presence, and the radius of burned organics is ~ 11.6 cm, consistent with observations on Mottarone granite [Elmi et al., 2017]. On the other hand, the quartz annealing temperature is reached inside a region of radius ~ 8 cm. The total increase in the internal energy of the rock is

$$\Delta U = \frac{1}{2} \rho C \left[\int_{R < a/\sqrt{2}} (T_i - T_a) dV + \int_{R \geq a/\sqrt{2}} (T_i - T_a) dV \right] = \frac{2\sqrt{2} A I_0^2 t_m}{3 \pi \sigma a} = 1.2 \times 10^8 \text{ J}, \quad (19)$$

but a fraction of the overall energy of the lightning, $\sim 10^9 - 10^{10}$ J [Rakov and Uman, 2007].

2.2.2. Cooling of the Rock by Convective Heat Loss at the Surface

After the lightning strike, the temperature in the rock evolves from T_i by heat conduction and convective heat loss at the surface. The governing equation of cooling is

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T \quad (20)$$

with the boundary and initial conditions

$$T \Big|_{r,z \rightarrow 0} \Rightarrow \text{finite}, \quad T \Big|_{r,z \rightarrow \infty} = T_a, \quad \kappa \rho C \frac{\partial T}{\partial z} \Big|_{z=0} = Q_v + Q_r, \quad T \Big|_{t=0} = T_a. \quad (21)$$

Because the surface heat loss rate $\sigma_{SB} T^4 / (\rho C \Delta r) \gg \kappa \nabla^2 T \sim \kappa (T - T_a) / \Delta r^2$, the surface quenches quickly and barely any heat is conducted from the hot strike center to the cooler periphery. Therefore, the peak temperature in the rock is reached right after the lightning strike stops, after which the temperature on the surface keeps dropping until the surface solidifies.

3. Discussion

We assume that the air can be treated as an ideal gas, and the ratio of specific heats γ remains constant, which may be unrealistic because the ionization and dissociation occurring at high temperature will change the apparent value of γ . As *Taylor* [1950b] pointed out, however, when comparing the same model as ours against the data from the first atomic explosion in New Mexico, the intense radiation from the center of the explosion can cause the apparent value of γ to rise, while ionization and dissociation at very high temperatures can decrease γ . The overall effect is that the whole system may behave as though the value of γ remains unchanged. The lightning bolt is hot ionized plasma, and the lightning channel, where strong ionization occurs, is very narrow, which suggests that radiation and ionization, facilitated by extremely high temperatures, are confined to much smaller regions than in atomic explosions. As a result, we expect that the deviation of the state of the air from ideality away from the lightning channel is insignificant.

We modeled the pressure exerted on a granite surface, and the temperature evolution in the rock, due to a lightning strike. The pressure model predicts a maximum pressure of ~ 7 GPa, smaller than the pressure thought to be required to generate shocked quartz, ~ 10 GPa. One possible reason for this discrepancy is that the energy density of the lightning channel is higher (> 10 MJ/m) than the assumed value of 3.3 MJ/m suggested by *Jones et al.* [1968]. There exists a large variation in estimates of the energy density of lightning. Our model gives an indirect way to quantify a lower bound for the lightning energy density. In the previous sections, our models rely on two basic parameters of the lightning, the energy density E and the lightning current I . These two parameters should be related; however, the details of that relationship remain unclear because of the difficulty in measuring them. Based on our model, if a lightning bolt strikes granite, leaving shock lamellae on its surface after cooling down, it is possible to provide some constraint on this relationship.

The formation of shock lamellae requires that, on the rock surface, the pressure disturbance at a distance r from the hit point is greater than $\Delta p \sim 10$ GPa [*Gieré et al.*, 2015; *Langenhorst and Deutsch*, 1994; *Ende et al.*, 2012]

$$\frac{2E}{Br^2(\gamma + 1)} > \Delta p, \quad (22)$$

while the temperature must be lower than the quartz annealing temperature T_q for the shock lamellae to survive

$$\Delta T = T_q - T_a > \frac{A l_0^2 t_m}{4\pi^2 \rho C \sigma r^4}. \quad (23)$$

The result is that

$$\frac{E}{I_0} > \frac{\Delta p B (\gamma + 1)}{4\pi} \sqrt{\frac{A t_m}{\rho C \sigma \Delta T}}. \quad (24)$$

For the annealing temperature $T_q = 1200^\circ\text{C}$, this ratio is 1.53×10^4 J/(A m). Therefore, if the peak current of the lightning is in the range 2–30 kA, the energy density is 30–460 MJ/m. Although the upper limit appears to be too large, the lower limit lies in the 10–50 MJ/m range proposed by *Collins et al.* [2012], which would yield a shock pressure greater than 7 GPa. The large upper limit can be attributed to the high temperature caused by intense Joule heating. There are several possible mechanisms that can decrease the temperature and the size of the annealed region, thus allowing for the survival of the PDFs.

Our model of temperature evolution in the rock yields high temperatures ($>10^5\text{K}$) near the strike point, assuming constant thermal and electrical properties. In reality, however, a decrease in moisture content with heating reduces the electrical conductivity, whereas after the temperature reaches the solidus of the rock, the increased electron activity in the molten rock greatly enhances electrical conductivity [Olhoeft, 1981; Roberts and Tyburczy, 1999], reducing the electrical heating effect. Also, near the lightning strike point, the modeled electric field is greater than 10^{11} V/m , and dielectric breakdown will occur. However, microstructural observations [Elmi *et al.*, 2017] confirm that these processes mainly take place at the rock surface, and the $\sim R^{-4}$ decay of temperature and $\sim R^{-2}$ decay of the electric field ensure that the material alteration is mostly confined to the close vicinity of the strike point. Our model predicts a radius of the burned region (11.6 cm) that is consistent with observed sizes of burned areas on fulgurite samples from Mottarone [Elmi *et al.*, 2017]. Thus, neglecting these variations in thermal and electrical properties with temperature and field strength does not change the overall behavior of our model. Also as stated above, the Mottarone granite has an apparent weathered layer $< 1\text{ mm}$ in thickness, but the weathering may actually extend much deeper into the rock, and associated fluid-filled cracks can increase the overall heat capacity C and even more significantly, the electrical conductivity σ . Olona *et al.* [2010] suggested that the conductivity of weathered granite can be an order of magnitude higher than that of pristine granite, which would reduce the temperature by an order of magnitude according to equation (18), and the radii of the areas prone to annealing of quartz, melting of granite, and burning of organics will shrink by a factor of $\sim 10^{1/4}$.

We also neglect the coupling between the time-varying electric and magnetic fields. We adopted a pulse-form current with a simple frequency spectrum, but real lightning events have a wide frequency range from $<300\text{ kHz}$ to $>30\text{ MHz}$ [Lan *et al.*, 2011]. Bowler [2004] solved the frequency-dependent electric field distribution in a homogeneous half-space, and the results show that the field strength has an exponentially decaying component with r and z , and consequently, electrical heating decreases faster than R^{-4} as in our model. These two effects—the increase in electrical conductivity in the weathered granite and coupling of the electric and magnetic fields—may lower the temperature rise during the lightning strike compared to our model predictions. This would result in a smaller radius of material around the strike point within which the annealing temperature of quartz is obtained, ensuring survival of the shock features.

Rock fulgurites are different from sand fulgurites, in which air fills the pore spaces in the sand ($\sim 40\%$ porosity for a random loose packing scenario) and acts as an insulator for the current. In sand fulgurites, heating done by the current only exists where dielectric breakdown occurs. In comparison, the electrical conductivity of granite is 10^{-3} S/m , much higher than that of air ($\sim 10^{-15}\text{ S/m}$), thus the current is able to pass through the rock and induce significant heating. Without pervasive electrical heating, thermal diffusion is the major mechanism for the formation of the melt tube [Carter *et al.*, 2010b; Pasek *et al.*, 2012] with a radius $\sim 3\text{ cm}$ (Appendix C), consistent with observed sand fulgurite tubes [Rogers, 1946; Gailliot, 1980; Pye, 1982; Essene and Fisher, 1986; Navarro-González *et al.*, 2007], but much smaller than the fulgurite region on rocks from Les Pradals [Gieré *et al.*, 2015] and Mount Mottarone [Elmi *et al.*, 2017].

Previous samples collected from the granite surface at Les Pradals, France indicate that shock lamellae are only observed in a surficial layer of the target $< 3\text{ }\mu\text{m}$ in thickness [Gieré *et al.*, 2015]. For an ideal flat rock surface, the main flash moves at $c_i \gg U$, and the angle between the propagation direction of the shock wave and the surface is $\theta \sim U/c_i \sim 10^{-6}$. The small angle ensures that for a quartz crystal on the surface of the rock, the vertical force exerted is only $\sim 10^{-6}$ of the horizontal force, and the pressure disturbance can only affect a thin layer on the surface before being significantly damped. This may explain why shocked quartz has only been observed at depths much smaller than the millimeter-scale topographic fluctuations. In reality the lightning may not strike at exactly 90° to the surface, and the rock surface is not flat. From equation (5), the disturbed pressure $p_1/p_0 \approx 2\gamma M^2/(\gamma + 1)$, where $M = U/c_s > 1$ is the Mach number. The Mach number will be higher for supersonic air flow passing a convex corner, which leads to higher pressure near topographic depressions. But even in that case, the pressure disturbance is still limited to a shallow depth near the rock surface.

The lightning strike can introduce hot air or gaseous reaction products into the molten rock which are then preserved as bubbles. Air inclusions may contain traces of ancient air which may be helpful in understanding paleoclimate [Navarro-González *et al.*, 2007]. High temperatures can also vaporize water and alkalis at the rock surface, and the burning of organic material generates NO_x and CO_x gases [Elmi *et al.*, 2017], facilitating the formation of a dense population of bubbles, preserved during quenching and responsible for the porous nature of rock fulgurites.

Table 2. Calculated Values of Physical Parameters for Granite Fulgurites

Calculated result	Value
Radius of burned region	11.6 cm
Radius of fulgurite region	9.3 cm
Minimum radius for PDF survival	8 cm
Heating energy	1.2×10^8 J
Maximum temperature	$\sim 10^5$ K
Maximum field strength	$\sim 10^{11}$ V/m
Shock pressure	7 GPa (for $E = 3.3$ MJ/m) to 21 GPa (for $E = 10$ MJ/m)
Shock inclination angle	$\sim 10^{-6}$ rad
Calculated energy density of lightning	30–460 MJ/m
Minimum ratio of energy density to current	1.53×10^4 J/(A m)

4. Conclusion

When a lightning bolt strikes a granite target, the shock wave sweeps the rock surface, and the distributed high current heats the rock. The shock wave creates shock lamellae in the substrate near the rock surface, and the electrical heating vaporizes material at the strike point, melts the granite (which is later quenched to form fulgurite), and burns organic material on a larger area of the surface. Our temperature and pressure models demonstrate that based on the observations of the spatial distribution of shocked quartz and burned organic material, we can constrain the energy and current intensity of the lightning event. We summarize the modeled parameter values for these and other physical parameters for granite fulgurites in Table 2. Our results further demonstrate that the presence of shock features in rocks cannot be taken as *prima facie* evidence for meteorite impacts.

Appendix A: Self-Similar Solution of the Cylindrical Shock Wave

Let the normalized location $\eta = r/R(t)$, and the pressure, density, and air velocity of the shock wave are assumed to be related to the undisturbed air as

$$\frac{p}{p_0} = \frac{U^2}{c_s^2} f(\eta), \quad \frac{\rho}{\rho_0} = \psi(\eta), \quad \frac{u}{U} = \phi(\eta). \quad (\text{A1})$$

Then equations (1) to (3) become

$$\phi'(\eta - \phi) = \frac{f'}{\gamma\psi} - \phi \quad (\text{A2})$$

$$\frac{\psi'}{\psi} = \frac{\phi' + \phi/\eta}{\eta - \phi} \quad (\text{A3})$$

$$2f + \eta f' + \frac{\gamma\psi'}{\psi} f(\phi - \eta) = f' \phi \quad (\text{A4})$$

with the boundary conditions

$$f(1) \approx \frac{2\gamma}{\gamma+1}, \quad \phi(1) \approx \frac{2}{\gamma+1}, \quad \psi(1) \approx \frac{\gamma+1}{\gamma-1}. \quad (\text{A5})$$

The system of ordinary differential equations (A2) to (A4) can then be solved numerically. The shock radius R is determined by

$$\begin{aligned} E &= 2\pi \int_0^R \rho \left(C_v T + \frac{u^2}{2} \right) r dr = 2\pi \int_0^R \left(\frac{p}{\gamma-1} + \frac{\rho u^2}{2} \right) r dr \\ &= \pi \rho_0 U^2 R^2 \int_0^1 \left[\frac{2f}{\gamma(\gamma-1)} + \psi \phi^2 \right] \eta d\eta = B \rho_0 U^2 R^2, \end{aligned} \quad (\text{A6})$$

where $B = 3.94$.

It is possible to apply the method to a spherically symmetric point explosion, where the energy release E' is related to the shock wave speed U and the shock radius R as

$$E' = 5.33\rho_0 U^2 R^3. \quad (A7)$$

The pressure disturbance therefore is

$$p = 0.156E'R^{-3}. \quad (A8)$$

Appendix B: Current Injection With Finite Width in Homogeneous Medium

When the medium is homogeneous, the governing equation for the electric potential in the cylindrical coordinate system is

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) \phi = 0 \quad (B1)$$

with the boundary conditions

$$\phi \Big|_{r=0} \Rightarrow \text{finite}, \quad \phi \Big|_{r,z \rightarrow \infty} = 0. \quad (B2)$$

The solving process is a routine separation of variables, and the general solution is

$$\phi = \int_0^\infty C_\lambda J_0(\lambda r) \exp(-\lambda z) d\lambda, \quad (B3)$$

where J_0 is the Bessel function of first kind with order zero. The current density at the surface is

$$-\sigma \frac{\partial \phi}{\partial z} \Big|_{z=0} = \sigma_0 \int_0^\infty C_\lambda \lambda J_0(\lambda r) d\lambda = \frac{I}{\pi a^2} H(a-r). \quad (B4)$$

where $H(x)$ is the Heaviside step function. Using a Hankel transform

$$a \int_0^\infty J_1(\lambda a) J_0(\lambda r) d\lambda = H(a-r) \quad (B5)$$

the solution of the potential is

$$\phi = \frac{I}{\pi a \sigma} \int_0^\infty \frac{\exp(-\lambda z)}{\lambda} J_1(\lambda a) J_0(\lambda r) d\lambda. \quad (B6)$$

As Bowler [2004] pointed out, when the channel radius $a \ll r$,

$$\lim_{a \rightarrow 0} \frac{J_1(\lambda a)}{\lambda a} = \frac{1}{2} \quad (B7)$$

so

$$\lim_{a \rightarrow 0} \phi = \frac{I}{2\pi\sigma} \int_0^\infty \exp(-\lambda z) J_0(\lambda r) d\lambda = \frac{I}{2\pi\sigma \sqrt{r^2 + z^2}} \quad (B8)$$

which is the point source potential field. However, the approximation has singularities for the potential and electric field at $r, z \rightarrow 0$, while direct evaluation of the electric field at the surface gives

$$\begin{aligned} \phi(r, z=0) &= \frac{I}{\pi a \sigma} \int_0^\infty \frac{1}{\lambda} J_1(\lambda a) d\lambda = \frac{I}{\pi a \sigma} \\ E_z(r, z=0) &= \frac{I}{\pi a \sigma} \int_0^\infty J_1(\lambda a) d\lambda = \frac{I}{\pi a^2 \sigma} \\ E_r(r, z=0) &= 0. \end{aligned} \quad (B9)$$

Therefore, the current density, which is the current intensity per unit surface area, has two different approximations in the vicinity of the origin, depending on whether we treat the “area” of the small region as a circle or as a half sphere.

Appendix C: Thermal Diffusion in Sand Fulgurites

For sand fulgurites, thermal diffusion is dominant. Carter *et al.* [2010b] and Pasek *et al.* [2012] treated sand fulgurites as generated by a line source, and the temperature evolution in the cylindrical coordinate system is

$$\kappa \nabla^2 T = \frac{\partial T}{\partial t}. \quad (C1)$$

For the ideal case where the line source is instantaneous, and the temperature within the lightning channel radius a is the peak temperature T_e , the solution is [Carslaw and Jaeger, 1959]

$$T = T_a + \frac{a^2(T_e - T_a)}{4\kappa t} \exp\left(-\frac{r^2}{4\kappa t}\right). \quad (C2)$$

If we set the temperature to the melting temperature of quartz sand $\sim 1650^\circ\text{C}$, the maximum radius of molten sand from the lightning path is about 3 cm.

Acknowledgments

We would like to express our gratitude to H.J. Melosh and one anonymous reviewer, whose thoughtful comments and criticisms greatly improved our paper, and to the Editor Andrew Newman for highlighting our paper with a Commentary by Melosh. We especially want to thank H.J. Melosh for his engaging Commentary on our paper, which amplified our conclusions and brought our work to the attention of a broader audience. Adopted parameter values in this paper are from references cited herein, and the calculated data used in this work are the output of the theoretical model described in the paper.

References

- Alvarez, W., P. Claeys, and S. W. Kieffer (1995), Emplacement of Cretaceous-Tertiary boundary shocked quartz from Chicxulub crater, *Science*, 269(5226), 930–935.
- Bohor, B. F., E. E. Foord, P. J. Modreski, and D. M. Triplehorn (1984), Mineralogic evidence for an impact event at the Cretaceous-Tertiary boundary, *Science*, 224(4651), 867–869.
- Bowler, N. (2004), Analytical solution for the electric field in a half space conductor due to alternating current injected at the surface, *J. Appl. Phys.*, 95(1), 344–348.
- Brocklesby, J. (1869), *Elements of Meteorology: With Questions for Examination, Designed for Schools and Academies*, Sheldon & Co., New York.
- Carslaw, H. S., and J. C. Jaeger (1959), *Conduction of Heat in Solids*, 2nd ed., Oxford Univ. Press, New York.
- Carter, E. A., M. D. Hargreaves, T. P. Kee, M. A. Pasek, and H. G. M. Edwards (2010a), A Raman spectroscopic study of a fulgurite, *Philos. Trans. R. Soc. London, Ser. A*, 368(1922), 3087–3097.
- Carter, E. A., M. A. Pasek, T. Smith, T. P. Kee, P. Hines, and H. G. M. Edwards (2010b), Rapid Raman mapping of a fulgurite, *Anal. Bioanal. Chem.*, 397(7), 2647–2658.
- Collins, G. S., H. J. Melosh, and M. A. Pasek (2012), Can lightning strikes produce shocked quartz?, in *Proceedings of the 43rd Lunar and Planetary Science Conference, Lunar and Planetary Inst. Technical Report*, vol. 43, p. 1160, LPI Contribution No. 1659, The Woodlands, Tex.
- Elmi, C., J. Chen, D. L. Goldsby, and R. Gieré (2017), Mineralogical and compositional features of rock fulgurites: A record of lightning effects on granite, *Am. Mineral.*, doi:10.2138/am-2017-5971.
- Ende, M., S. Schorr, G. Kloess, A. Franz, and M. Tovar (2012), Shocked quartz in Sahara fulgurite, *Eur. J. Mineral.*, 24(3), 499–507.
- Eppelbaum, L. I. Kutasov, and A. Pilchin (2014), *Applied Geothermics*, Lecture Notes in Earth Syst. Sci., Springer, Berlin.
- Essene, E. J., and D. C. Fisher (1986), Lightning strike fusion: Extreme reduction and metal-silicate liquid immiscibility, *Science*, 234(4773), 189–193.
- Gailliot, M. P. (1980), “Petrified lightning” a discussion of sand fulgurites, *Rocks Miner.*, 55(1), 13–17.
- Gieré, R., W. Wimmenauer, H. Müller-Sigmund, R. Wirth, G. R. Lumpkin, and K. L. Smith (2015), Lightning-induced shock lamellae in quartz, *Am. Mineral.*, 100(7), 1645–1648.
- Goltrant, O., H. Leroux, J.-C. Doukhan, and P. Cordier (1992), Formation mechanisms of planar deformation features in naturally shocked quartz, *Phys. Earth Planet. Inter.*, 74(3–4), 219–240.
- Grapes, R. H., and H. Müller-Sigmund (2010), Lightning-strike fusion of gabbro and formation of magnetite-bearing fulgurite, Cornone di Blumone, Adamello, Western Alps, Italy, *Mineral. Petrol.*, 99(1), 67–74.
- Grieve, R. A. F., F. Langenhorst, and D. Stöffler (1996), Shock metamorphism of quartz in nature and experiment: II. Significance in geoscience, *Meteorit. Planet. Sci.*, 31(1), 6–35.
- Jones, D. L., G. G. Goyer, and M. N. Plooster (1968), Shock wave from a lightning discharge, *J. Geophys. Res.*, 73(10), 3121–3127.
- Lan, Y., Y. Jun Zhang, W. Sheng Dong, W. tao Lu, H. Yi Liu, and D. Zheng (2011), Broadband analysis of chaotic pulse trains generated by negative cloud-to-ground lightning discharge, *J. Geophys. Res.*, 116, D17109, doi:10.1029/2010JD015159.
- Lang, T. J., et al. (2016), WMO world record lightning extremes: Longest reported flash distance and longest reported flash duration, *Bull. Am. Meteorol. Soc.*, 98(6), 1153–1168.
- Langenhorst, F., and A. Deutsch (1994), Shock experiments on pre-heated α - and β -quartz: I. optical and density data, *Earth Planet. Sci. Lett.*, 125(1–4), 407–420.
- Lin, S.-C. (1954), Cylindrical shock waves produced by instantaneous energy release, *J. Appl. Phys.*, 25(1), 54–57.
- Marsh, K. N. (1987), *Recommended Reference Materials for the Realization of Physicochemical Properties*, Blackwell Sci. Publ., Oxford, U. K.
- Kumazaki, K., K. Nakamura, K. Naito, and K. Horii (1993), Production of artificial fulgurite by utilizing rocket triggered lightning, in *Proc. 8th Int. Symp. on High Voltage Engineering*, pp. 269–72, Yokohama, Japan.
- Navarro-González, R., S. A. Mahan, A. K. Singhvi, R. Navarro-Aceves, J.-L. Rajot, C. P. McKay, P. Coll, and F. Raulin (2007), Paleocology reconstruction from trapped gases in a fulgurite from the late Pleistocene of the Libyan desert, *Geology*, 35(2), 171–174.
- Olhoeft, G. R. (1981), Electrical properties of granite with implications for the lower crust, *J. Geophys. Res.*, 86(B2), 931–936.
- Olona, J., J. A. Pulgar, G. F. Fernández-Viejo, C. L. López-Fernández, and J. M. G. González-Cortina (2010), Weathering variations in a granitic massif and related geotechnical properties through seismic and electrical resistivity methods, *Near Surf. Geophys.*, 8, 585–599.

- Pasek, M. A., and M. Hurst (2016), A fossilized energy distribution of lightning, *Sci. Rep.*, 6, 30586.
- Pasek, M. A., K. Block, and V. Pasek (2012), Fulgurite morphology: A classification scheme and clues to formation, *Contrib. Mineral. Petrol.*, 164(3), 477–492.
- Plooster, M. N. (1971), Numerical model of the return stroke of the lightning discharge, *Phys. Fluids*, 14(10), 2124–2133.
- Pye, K. (1982), SEM observations on some sand fulgurites from northern Australia, *J. Sediment. Res.*, 52(3), 991–998.
- Rakov, V. A., and M. A. Uman (2007), *Lightning: Physics and Effects*, Cambridge Univ. Press, Cambridge, U. K.
- Roberts, J. J., and J. A. Tyburczy (1999), Partial-melt electrical conductivity: Influence of melt composition, *J. Geophys. Res.*, 104(B4), 7055–7065.
- Robertson, E. C. (1988), *Thermal Properties of Rocks*, U.S. Dep. of the Inter., Geol. Surv., Reston, Va.
- Rogers, A. F. (1946), Sand fulgurites with enclosed lechatelierite from Riverside county, California, *J. Geol.*, 54(2), 117–122.
- Taylor, G. (1950a), The formation of a blast wave by a very intense explosion. I. Theoretical discussion, *Proc. R. Soc. London, Ser. A*, 201(1065), 159–174.
- Taylor, G. (1950b), The formation of a blast wave by a very intense explosion. II. The atomic explosion of 1945, *Proc. R. Soc. London, Ser. A*, 201(1065), 175–186.
- Uman, M. A. (1964), The peak temperature of lightning, *J. Atmos. Terr. Phys.*, 26(1), 123–128.
- Uman, M. A., and E. P. Krider (1989), Natural and artificially initiated lightning, *Science*, 246(4929), 457–464.
- White, R. H., and M. A. Dietenberger (2001), Wood products: Thermal degradation and fire, in *The Encyclopedia of Materials: Science and Technology*, edited by K. H. J. Buchow et al., pp. 9712–9716, Elsevier, Amsterdam.
- Williams, D. J., and W. Johnson (1980), A note on the formation of fulgurites, *Geol. Mag.*, 117(3), 293–296.